

# Logarithmic correction to the Cardy–Verlinde formula in Achúcarro–Ortiz black hole

M.R. Setare<sup>a</sup>

Physics Department, Institute for Studies in Theoretical Physics and Mathematics (IPM), P.O. Box 19395-5531, Tehran, Iran

Received: 11 October 2003 / Revised version: 8 January 2004 /

Published online: 25 February 2004 – © Springer-Verlag / Società Italiana di Fisica 2004

**Abstract.** In this paper we calculate the leading-order correction due to small statistical fluctuations around equilibrium to the Bekenstein–Hawking entropy formula for the Achúcarro–Ortiz black hole, which is the most general two-dimensional black hole derived from the three-dimensional rotating Bañados–Teitelboim–Zanelli black hole. Then we obtain the same correction to the Cardy–Verlinde entropy formula (which is supposed to be an entropy formula of conformal field theory in any number of dimensions).

## 1 Introduction

It is commonly believed that any valid theory of quantum gravity must necessarily incorporate the Bekenstein–Hawking definition of the black hole entropy [1,2] into its conceptual framework. However, the microscopic origin of this entropy remains an enigma for two reasons. First of all, although the various counting methods have pointed to the expected semi-classical result, there is still a lack of recognition as to what degrees of freedom are truly being counted. This ambiguity can be attributed to most of these methods being based on dualities with simpler theories, thus obscuring the physical interpretation from the perspective of the black hole in question. Secondly, the vast and varied number of successful counting techniques only serve to cloud up an already fuzzy picture.

The de Sitter/conformal field theory correspondence (dS/CFT) [3–31] may hold the key to its microscopical interpretation. Naively, we would expect the dS/CFT correspondence to proceed along the lines of anti-de Sitter/conformal field theory (AdS/CFT) correspondence [32] because the de Sitter spacetime can be obtained from an anti-de Sitter spacetime by analytically continuing the cosmological constant to imaginary values. The Cardy–Verlinde formula, proposed by Verlinde [33], relates the entropy of a certain CFT with its total energy and its Casimir energy in arbitrary dimensions. Using the  $\text{AdS}_d/\text{CFT}_{d-1}$  and  $\text{dS}_d/\text{CFT}_{d-1}$  correspondences, this formula has been shown to hold exactly for different black holes. In a previous paper [28], by using the Cardy–Verlinde formula, we have obtained the entropy of the Achúcarro–Ortiz black hole which is a two-dimensional black hole derived from the three-dimensional rotating BTZ black hole.

In 1992 Bañados, Teitelboim and Zanelli (BTZ) [34, 35] showed that  $(2 + 1)$ -dimensional gravity has a black hole solution. This black hole is described by two (gravitational) parameters, the mass  $M$  and the angular momentum (spin)  $J$ . It is locally AdS and thus it differs from the Schwarzschild and Kerr solutions since it is an asymptotically anti-de Sitter instead of a flat spacetime. Additionally, it has no curvature singularity at the origin. AdS black holes are members of this two-parametric family of BTZ black holes, and they are very interesting in the framework of string theory and black hole physics [36, 37].

For systems that admit 2D CFTs as duals, the Cardy formula [38] can be applied directly. This formula gives the entropy of a CFT in terms of the central charge  $c$  and the eigenvalue of the Virasoro operator  $l_0$ . However, it should be pointed out that this evaluation is possible as soon as one has explicitly shown (e.g. using the  $\text{AdS}_d/\text{CFT}_{d-1}$  correspondence) that the system under consideration is in correspondence with a 2D CFT [39,40].

In [39] Cadoni and Mignemi, using the Cardy formula, have calculated the statistical entropy of the two-dimensional Jackiw–Teitelboim black hole, which can be considered as the dimensional reduction of the  $j = 0$  (zero angular momentum) BTZ black hole. Using a canonical realization of the asymptotic symmetry of two-dimensional anti-de Sitter space and Cardy’s formula, they have calculated the statistical entropy of a 2D black hole. In this case this reference relates a two-dimensional black hole to a one-dimensional CFT, living on the boundary of  $\text{AdS}_2$ . In fact the one-dimensional nature of the boundary CFT implies that we are dealing with some kind of particle quantum mechanics, rather than quantum field theory. On the other hand, as has been shown in second paper by Cadoni and Mignemi [39], in the family of the  $\text{AdS}_d/\text{CFT}_{d-1}$  dualities, the  $d = 2$  case is very similar to the  $d = 3$  one, the

<sup>a</sup> e-mail: rezakord@ipm.ir

conformal group being in both instances infinite dimensional. But a feature that is peculiar to the  $d = 2$  case is the complete equivalence of the diffeomorphisms and the conformal group in one dimension. The physical implication of this equivalence is that the usual difference between gauge symmetries and symmetries related to conserved charges disappears. If one accepts that the  $d = 2$  case is not fundamental then the  $\text{CFT}_1$  should be thought of just as (half) of  $\text{CFT}_2$ , in the way explained in [39]; therefore in the  $d = 2$  context the general  $\text{AdS}_d/\text{CFT}_{d-1}$  duality becomes a duality between two 2D conformal field theories. Although the possibility of describing 2D black holes by means of a CFT has been widely investigated [39, 41–43], it is not completely clear if it is always possible to mimic the gravitational dynamics of the 2D black hole through a CFT. However, in some cases and/or for generic black holes in particular regimes, CFTs have been shown to give a good description; this is in particular true for black holes in AdS space. In this paper I consider the two-dimensional Achúcarro–Ortiz black hole which is asymptotically  $\text{AdS}_2$ , then I show a 2D CFT to give a good description in this case also. When the black hole is put in correspondence with a 2D CFT, the entropy of the black hole horizon is reproduced using the Cardy–Verlinde formula.

There has been much interest recently in calculating the quantum corrections to  $S_{\text{BH}}$  (the Bekenstein–Hawking entropy) [44–80]. The leading-order correction is proportional to  $\ln S_{\text{BH}}$ . There are *two* distinct and separable sources for this logarithmic correction [71, 74] (see also the recent paper by Gour and Medved [78]). Firstly, there should be a correction to the number of microstates, which is a quantum correction to the microcanonical entropy; secondly, as any black hole will typically exchange heat or matter with its surroundings, there should also be a correction due to thermal fluctuations in the horizon area. In a recent work, Carlip [46] has deduced the leading-order quantum correction to the classical Cardy formula. The Cardy formula follows from a saddle-point approximation of the partition function for a two-dimensional conformal field theory. This leads to the theory’s density of states, which is related to the partition function by way of a Fourier transform [81]. In [66] Medved has applied Carlip’s formulation to the case of a generic model of two-dimensional gravity with coupling to a dilaton field.

In this paper we consider the Achúcarro–Ortiz black hole, In Sect. 2 we calculate the corresponding thermodynamical quantities for black hole horizon. In Sect. 3 we calculate the leading-order correction due to small statistical fluctuations around equilibrium to the Bekenstein–Hawking entropy formula, and then we obtain the same correction to the Cardy–Verlinde entropy formula. In the other term we assume the equality of the Bekenstein–Hawking and the CFT entropy; then we use the known statistical corrections to the Bekenstein–Hawking entropy to predict the corrections to the Cardy–Verlinde formula. The last section contains a summary of this paper.

## 2 Thermodynamical quantities of the Achúcarro–Ortiz black hole

The black hole solutions of Bañados, Teitelboim and Zanelli [34, 35] in  $(2+1)$  spacetime dimensions are derived from a three-dimensional theory of gravity:

$$S = \int dx^3 \sqrt{-g} \left( {}^{(3)}R + 2\Lambda \right), \quad (1)$$

with a negative cosmological constant ( $\Lambda = \frac{1}{l^2} > 0$ ).

The corresponding line element is

$$ds^2 = - \left( -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2} \right) dt^2 + \frac{dr^2}{\left( -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2} \right)} + r^2 \left( d\theta - \frac{J}{2r^2} dt \right)^2. \quad (2)$$

There are many ways to reduce the three-dimensional BTZ black hole solutions to the two-dimensional charged and uncharged dilatonic black holes [82, 83]. The Kaluza–Klein reduction of the  $(2+1)$ -dimensional metric (2) yields a two-dimensional line element:

$$ds^2 = -g(r)dt^2 + g(r)^{-1}dr^2, \quad (3)$$

where

$$g(r) = \left( -M + \frac{r^2}{l^2} + \frac{J^2}{4r^2} \right), \quad (4)$$

with  $M$  the Arnowitt–Deser–Misner (ADM) mass,  $J$  the angular momentum (spin) of the BTZ black hole and  $-\infty < t < +\infty$ ,  $0 \leq r < +\infty$ ,  $0 \leq \theta < 2\pi$ .

The outer and inner horizons, i.e.  $r_+$  (henceforth simply the black hole horizon) and  $r_-$  respectively, concerning the positive mass black hole spectrum with spin ( $J \neq 0$ ) of the line element (3) are given by

$$r_{\pm}^2 = \frac{l^2}{2} \left( M \pm \sqrt{M^2 - \frac{J^2}{l^2}} \right), \quad (5)$$

and therefore, in terms of the inner and outer horizons, the black hole mass and the angular momentum are given, respectively, by

$$M = \frac{r_+^2}{l^2} + \frac{J^2}{4r_+^2} \quad (6)$$

and

$$J = \frac{2r_+r_-}{l}, \quad (7)$$

with the corresponding angular velocity

$$\Omega = \frac{J}{2r_+^2}. \quad (8)$$

The Hawking temperature  $T_{\text{H}}$  of the black hole horizon is [84]

$$T_{\text{H}} = \frac{1}{2\pi r_+} \sqrt{\left( \frac{r_+^2}{l^2} + \frac{J^2}{4r_+^2} \right)^2 - \frac{J^2}{l^2}}$$

$$= \frac{1}{2\pi r_+} \left( \frac{r_+^2}{l^2} - \frac{J^2}{4r_+^2} \right). \tag{9}$$

In two spacetime dimensions we do not have an area law for the black hole entropy; however, one can use a thermodynamical reasoning to define the entropy [84]

$$S_{\text{BH}} = 4\pi r_+. \tag{10}$$

The specific heat of the black hole is given by

$$\begin{aligned} C &= \frac{dE}{dT} = \frac{dM}{dT} = 4\pi r_+ \left( \frac{r_+^2 - r_-^2}{r_+^2 + 3r_-^2} \right) \\ &= S_{\text{BH}} \left( \frac{r_+^2 - r_-^2}{r_+^2 + 3r_-^2} \right); \end{aligned} \tag{11}$$

if  $r_+ > r_-$ , then the above specific heat is positive. The stability condition is equivalent to the specific heat being positive, so that the corresponding canonical ensemble is stable.

### 3 Logarithmic correction to the Bekenstein–Hawking entropy and Cardy–Verlinde formula

There has been much interest recently in calculating the quantum corrections to  $S_{\text{BH}}$  (the Bekenstein–Hawking entropy) [44–80]. The corrected formula takes the form

$$S = S_0 - \frac{1}{2} \ln C + \dots \tag{12}$$

When  $r_+ \gg r_-$ ,  $C \simeq S_{\text{BH}} = S_0$  in which case we have

$$S = S_0 - \frac{1}{2} \ln S_0 + \dots \tag{13}$$

It is now possible to drive the corresponding correction to the Cardy–Verlinde formula. In a recent paper, Verlinde [33] proposed a generalization of the Cardy formula, which holds for the  $(1 + 1)$  dimensional conformal field theory (CFT), to  $(n + 1)$ -dimensional spacetime described by the metric

$$ds^2 = -dt^2 + R^2 d\Omega_n \tag{14}$$

where  $R$  is the radius of a  $n$ -dimensional sphere.

The generalized Cardy formula (hereafter named the Cardy–Verlinde formula) is given by

$$S_{\text{CFT}} = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)}, \tag{15}$$

where  $E$  is the total energy,  $E_C$  is the Casimir energy, and  $a$  and  $b$  are a priori arbitrary positive coefficients, independent of  $R$  and  $S$ . The definition of the Casimir energy is derived by the violation of the Euler relation as

$$E_C \equiv n(E + pV - TS - J\Omega), \tag{16}$$

where the pressure of the CFT is defined as  $p = E/nV$ . The total energy may be written as the sum of two terms

$$E(S, V) = E_E(S, V) + \frac{1}{2} E_C(S, V), \tag{17}$$

where  $E_E$  is the purely extensive part of the total energy  $E$ . The Casimir energy  $E_C$  as well as the purely extensive part of the energy  $E_E$  expressed in terms of the radius  $R$  and the entropy  $S$  are written as

$$E_C = \frac{b}{2\pi R} S^{1-\frac{1}{n}}, \tag{18}$$

$$E_E = \frac{a}{4\pi R} S^{1+\frac{1}{n}}. \tag{19}$$

After the work of Witten on the  $\text{AdS}_d/\text{CFT}_{d-1}$  correspondence [85], Savonije and Verlinde proved that the Cardy–Verlinde formula (15) can be derived using the thermodynamics of AdS–Schwarzschild black holes in an arbitrary number of dimensions [86]. For the present discussion, the total entropy is assumed to be of the form (13), where the uncorrected entropy,  $S_0$ , corresponds to the associated one in (10). Since the two-dimensional Achúcarro–Ortiz black hole is asymptotically anti-de Sitter, the total energy is  $E = M$ . It then follows by employing (6)–(9) that the Casimir energy (16) can be expressed in term of the uncorrected entropy:

$$E_C = \frac{J^2}{2r_+^2} - \frac{1}{2} T_H \ln S_0, \tag{20}$$

Then by setting the above corrected Casimir energy in (15) and expanding in terms of  $\frac{1}{2} T_H \ln S_0$  we obtain

$$\begin{aligned} &\frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)} \\ &\simeq S_0 \left( 1 + \frac{1}{2} T_H \ln S_0 \frac{E_C - E}{E_C (2E - E_C)} \right). \end{aligned} \tag{21}$$

In the limit where the correction is small, the coefficient of the logarithmic term on the right-hand side of (21) can be expressed in terms of the energy and the Casimir energy:

$$\frac{(E_C - E)}{2E_C (2E - E_C)} T_H S_0 = \frac{(E_C - E)(2E - E_C - E_q)}{2E_C (2E - E_C)}, \tag{22}$$

where

$$E_q = Q\phi = J\Omega \tag{23}$$

is the electromagnetic energy; in our analysis the charge  $Q$  is the angular momentum  $J$  of the two-dimensional Achúcarro–Ortiz black hole, and the corresponding electric potential  $\phi$  is the angular velocity  $\Omega$ . We may conclude, therefore, that, in the limit where the logarithmic corrections are sub-dominant, (21) can be rewritten to express the entropy in terms of the energy and the Casimir energy. We have

$$S_0 = \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C (2E - E_C)} \tag{24}$$

$$- \frac{(E_C - E)(2E - E_C - E_q)}{2E_C(2E - E_C)} \times \ln \left( \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C(2E - E_C)} \right),$$

and consequently, the total entropy (13), to first order in the logarithmic term, is given by

$$S \simeq \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C(2E - E_C)} - \left( \frac{(E_C - E)(2E - E_C - E_q)}{2E_C(2E - E_C)} + \frac{1}{2} \right) \times \ln \left( \frac{2\pi R}{\sqrt{ab}} \sqrt{E_C(2E - E_C)} \right). \quad (25)$$

Therefore, taking into account thermal fluctuations defines the logarithmic corrections to the black hole entropies. As a result the Cardy–Verlinde formula receives logarithmic corrections in our Achúcarro–Ortiz black hole background in two dimensions, in a way similar to the Cardy–Verlinde formula for the SAdS and SdS black holes in five dimensions [77, 79] also for TRNdS black holes in any number of dimensions [80]. It is easily seen that the logarithmic prefactor is negative and therefore the thermal corrections are also negative. Furthermore, the entropy of the two-dimensional Achúcarro–Ortiz black hole described in the context of the analysis by Das et al. [47] by the modified Cardy–Verlinde formula satisfies the holographic bound [87].

## 4 Conclusion

For a large class of black holes, the Bekenstein–Hawking entropy formula receives additive logarithmic corrections due to thermal fluctuations. On the basis of general thermodynamic arguments, Das et al. [47] deduced that the black hole entropy can be expressed as

$$\mathcal{S} = \ln \rho = S_0 - \frac{1}{2} \ln (C T^2) + \dots \quad (26)$$

In this paper we have analyzed this correction of the entropy of the Achúcarro–Ortiz black hole in two dimensions in the light of AdS/CFT. We have obtained the logarithmic correction to the black hole entropy. Then using the form of the logarithmic correction (13) we have derived the corresponding correction to the Cardy–Verlinde formula, which relates the entropy of a certain CFT to its total energy and Casimir energy. The result of this paper is that the CFT entropy can be written in the form (25).

## References

- J.D. Bekenstein, *Lett. Nuovo. Cim.* **4**, 737 (1972); *Phys. Rev. D* **7**, 2333 (1973); **9**, 3292 (1974)
- S.W. Hawking, *Comm. Math. Phys.* **25**, 152 (1972); J.M. Bardeen, B. Carter, S.W. Hawking, *Comm. Math. Phys.* **31**, 161 (1973)
- A. Strominger, *JHEP* **0110**, 034 (2001); M. Spradlin, A. Strominger, A. Volovich, *Les Houches Lectures on De Sitter Space*, hep-th/0110007
- M.-I. Park, *Phys. Lett. B* **440**, 275 (1998)
- C.M. Hull, *JHEP* **9807**, 021 (1998); **9811**, 017 (1998); C.M. Hull, R.R. Khuri, *Nucl. Phys. B* **536**, 219 (1998); **575**, 231 (2000)
- P.O. Mazur, E. Mottola, *Phys. Rev. D* **64**, 104022 (2001)
- V. Balasubramanian, P. Horava, D. Minic, *JHEP* **0105**, 043 (2001)
- M.-I. Park, *Nucl. Phys. B* **544**, 377 (1999)
- M. Li, *JHEP* **0204**, 005 (2002)
- S. Nojiri, S.D. Odintsov, *Phys. Lett. B* **519**, 145 (2001); *JHEP* **0112**, 033 (2001); S. Ogushi, *Phys. Rev. D* **65**, 023521 (2002); S. Nojiri, S.D. Odintsov, S. Ogushi, *Int. J. Mod. Phys. A* **17**, 4809 (2002)
- D. Klemm, A.C. Petkou, G. Siopsis, *Nucl. Phys. B* **601**, 380 (2001); D. Klemm, *Nucl. Phys. B* **625**, 295 (2002); S. Cacciatori, D. Klemm, *Class. Quant. Grav.* **19**, 579 (2002)
- Y.-h. Gao, *Symmetries, Matrices, and de Sitter Gravity*, hep-th/0107067
- J. Bros, H. Epstein, U. Moschella, *Phys. Rev. D* **65**, 84012 (2002)
- E. Halyo, *De Sitter Entropy and Strings*, hep-th/0107169
- A.J. Tolley, N. Turok, *Quantization of the massless minimally coupled scalar field and the dS/CFT correspondence*, hep-th/0108119
- T. Shiromizu, D. Ida, T. Torii, *JHEP* **0111**, 010 (2001)
- B. McInnes, *Nucl. Phys. B* **627**, 311 (2002)
- A. Strominger, *JHEP* **0111**, 049 (2001)
- V. Balasubramanian, J. de Boer, D. Minic, *Phys. Rev. D* **65**, 123508 (2002)
- Y.S. Myung, *Mod. Phys. Lett. A* **16**, 2353 (2001)
- B. Carneiro da Cunha, *Phys. Rev. D* **65**, 104025 (2002)
- U.H. Danielsson, *JHEP* **0203**, 020 (2002)
- A.M. Ghezelbash, R.B. Mann, *JHEP* **0201**, 005 (2002); A.M. Ghezelbash, D. Ida, R.B. Mann, T. Shiromizu, *Phys. Lett. B* **535**, 315 (2002)
- S. Ogushi, *Mod. Phys. Lett. A* **17**, 51 (2002)
- M.R. Setare, *Mod. Phys. Lett. A* **17**, 2089 (2002)
- M.R. Setare, R. Mansouri, *Int. J. Mod. Phys. A* **18**, 4443 (2003)
- M.R. Setare, M.B. Altaie, *Eur. Phys. J. C* **30**, 273 (2003)
- M.R. Setare, E.C. Vagenas, *Phys. Rev. D* **68**, 064014 (2003)
- A.J.M. Medved, *Class. Quant. Grav.* **19**, 919 (2002)
- M. Cadoni, P. Carta, S. Mignemi, *Nucl. Phys. B* **632**, 383 (2002)
- M. Cadoni, P. Carta, M. Cavaglià, S. Mignemi, *Phys. Rev. D* **66**, 065008 (2002)
- J. Maldacena, *Adv. Theor. Math. Phys.* **2**, 231 (1998); S.S. Gubser, I.R. Klebanov, A.M. Polyakov, *Phys. Lett. B* **428**, 105 (1998) [hep-th/9802109]; E. Witten, *Adv. Theor. Math. Phys.* **2**, 253 (1998)
- E. Verlinde, *On the Holographic Principle in a Radiation Dominated Universe*, hep-th/0008140
- M. Bañados, C. Teitelboim, J. Zanelli, *Phys. Rev. Lett.* **69**, 1849 (1992)
- M. Bañados, M. Henneaux, C. Teitelboim, J. Zanelli, *Phys. Rev. D* **48**, 1506 (1993)
- J. Maldacena, J. Michelson, A. Strominger, *JHEP* **9902**, 011 (1999)
- M. Spradlin, A. Strominger, *JHEP* **9911**, 021 (1999)

38. J.A. Cardy, Nucl. Phys. B **270**, 186 (1986)
39. M. Cadoni, S. Mignemi, Phys. Rev. D **59**, 081501 (1999); Nucl. Phys. B **557**, 165 (1999)
40. A. Strominger, JHEP **9802**, 009 (1998)
41. A. Strominger, J. High Energy Phys. **01**, 007 (1999); S. Cacciatori, D. Klemm, D. Zanon, Class. Quant. Grav. **17**, 1731 (2000)
42. M. Cadoni, M. Cavaglia, Phys. Rev. D **63**, 084024 (2001); Phys. Lett. B **499**, 315 (2001)
43. M. Cadoni, P. Carta, Phys. Lett. B **522**, 126 (2001)
44. R.K. Kaul, P. Majumdar, Phys. Rev. Lett. **84**, 5255 (2000)
45. J. Makela, P. Repo, gr-qc/9812075
46. S. Carlip, Class. Quant. Grav. **17**, 4175 (2000)
47. S. Das, P. Majumdar, R.K. Bhaduri, Class. Quant. Grav. **19**, 2355 (2002)
48. S.N. Solodukhin, Phys. Rev. D **51**, 609 (1995); D **51**, 618 (1995); D **57**, 2410 (1998); N.E. Mavromatos, E. Winstanley, Phys. Rev. D **53**, 3190 (1996)
49. D.V. Fursaev, Phys. Rev. D **51**, 5352 (1995)
50. C.O. Lousto, Phys. Lett. B **352**, 228 (1995)
51. V.P. Frolov, W. Israel, S.N. Solodukhin, Phys. Rev. D **54**, 2732 (1996)
52. R.B. Mann, S.N. Solodukhin, Phys. Rev. D **55**, 3622 (1997); Nucl. Phys. B **523**, 293 (1998)
53. H.A. Kastrup, Phys. Lett. B **413**, 267 (1997)
54. A.J.M. Medved, G. Kunstatter, Phys. Rev. D **60**, 104029 (1999)
55. G. Gour, Phys. Rev. D **61**, 021501 (2000)
56. O. Obregon, M. Sabido, V.I. Tkach, Gen. Rel. Grav. **33**, 913 (2001)
57. S. Das, R.K. Kaul, P. Majumdar, Phys. Rev. D **63**, 044019 (2001)
58. D. Birmingham, S. Sen, Phys. Rev. D **63**, 047501 (2001)
59. P. Majumdar, Pramana **55**, 511 (2000); hep-th/0110198
60. A.J.M. Medved, G. Kunstatter, Phys. Rev. D **63**, 104005 (2001)
61. S.A. Major, K.L. Setter, Class. Quant. Grav. **18**, 5125 (2001); **18**, 5293 (2001)
62. D. Birmingham, I. Sachs, S. Sen, Int. J. Mod. Phys. D **10**, 833 (2001)
63. T.R. Govindarajan, R.K. Kaul, V. Suneeta, Class. Quant. Grav. **18**, 2877 (2001)
64. M. Cavaglia, A. Fabbri, Phys. Rev. D **65**, 044012 (2002)
65. K.S. Gupta, S. Sen, Phys. Lett. B **526**, 121 (2002)
66. A.J.M. Medved, Class. Quant. Grav. **19**, 2503 (2002)
67. K.S. Gupta, hep-th/0204137
68. S. Mukherji, S.S. Pal, JHEP **0205**, 026 (2002)
69. S. Das, hep-th/0207072
70. A.J.M. Medved, hep-th/0210017, gr-qc/0211004
71. G. Gour, Phys. Rev. D **66**, 104022 (2002)
72. I.B. Khriplovich, gr-qc/0210108
73. R.K. Kaul, S.K. Rama, gr-qc/0301128
74. A. Chatterjee, P. Majumdar, gr-qc/0303030
75. S. Das, V. Husain, hep-th/0303089
76. M.M. Akbar, S. Das, hep-th/0304076
77. J.E. Lidsey, S. Nojiri, S.D. Odintsov, S. Ogushi, Phys. Lett. B **544**, 337 (2002)
78. G. Gour, A.J.M. Medved, gr-qc/0305018
79. S. Nojiri, S.D. Odintsov, S. Ogushi, hep-th/0212047
80. M.R. Setare, Phys. Lett. B **573**, 173 (2003)
81. S. Carlip, Class. Quant. Grav. **15**, 3609 (1998)
82. A. Achúcarro, M.E. Ortiz, Phys. Rev. D **48**, 3600 (1993)
83. D.A. Lowe, A. Strominger, Phys. Rev. Lett. **73**, 1468 (1994)
84. A. Kumar, K. Ray, Phys. Lett. B **351**, 431 (1995)
85. E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998)
86. I. Savonije, E. Verlinde, Phys. Lett. B **507**, 305 (2001)
87. G. 't Hooft, Dimensional Reduction in Quantum Gravity, gr-qc/9310026; L. Susskind, J. Math. Phys. **36**, 6377 (1995)